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## THE ACCURACY PREDICTION TECHNOLOGY BASED ON ROUGH SURFACE ANALYSIS METHOD

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### ABSTRACT

This article focuses on the precision assembly of aero crafts. And the digital assembly controlling technology is developed. In the research process, the accuracy prediction technology based on rough surface analysis method is proposed, and the theory of state space model based on frequency domain analysis method in assembly process is studied and established. The mathematical expression form is expressed.

### KEY WORDS

Rough surface analysis method, State space model and Angular variation.

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### INTRODUCTION

An important index of aero crafts cabin assembly process is angular variation among different axes, Due to the installation of the axis of the sensor inside the cabin is based on the axis of the cabin, to ensure the mechanical axial angular variation is the important method to ensure the angular variation of different electrical axes. And to ensure the angular variation of electrical axes is an important prerequisite to ensure the normal operation of the aero crafts. Therefore, through the assembly controlling method to ensure the angular variation of different cabins is very meaningful.

Assembly is an important part of the life cycle of products, the assembly quality and the assemblability of products directly influence and restrict the performance and cost of products. According to statistics, in the modern manufacturing,

the workload of the assembly process occupies 20%-70% of the entire product development workload, the average value can reach 45% and the assembly time accounted for 40% - 60% of the entire production time. At the same time, the assembly of the products is usually a large amount of manual labour, high cost and belongs to the rear end of the product development work. The benefit brought by improving the productivity of assembly and assembly precision is much more remarkable than that of simply reducing the parts production costs<sup>1</sup>.

At present, the assembly level of many products still strands in manual stage and experience stage, accuracy analysis and prediction before or the during the assembly process cannot be realized. The only way to verify the product whether meet the design index requirements or not is in the final system adjustment process, the result is to assemble a set of qualified products to be conducted several rounds of "assembly measurement-adjustment-measurement process, some products even need repeat disassembly and assembly to meet the design requirements, which not only consumes the large amount of manpower, financial resources, efficiency is very low. But also the features of parts and components are worn out because of the repeated disassembly and assembly, which seriously affects the assembly accuracy. Therefore, it is necessary to carry on the theoretical analysis and the prediction to the assembly accuracy, thus to raise the level of the assembly process level.

Structures of aero crafts are becoming more and more complicated nowadays. The number of cabins has also increased. The angular variation is an important as well as rigorous index in terms of the assembly of columned parts, which has become more obvious because of assembly variation propagation. In this sense, the assembly angular variation should be reduced for the sake of coherence, stability and reliability of aero crafts.

In large and complex multi-station assembly systems, there are many factors affecting product quality, among which, dimensional control has a significant impact on the overall product quality and performance, further more on the productivity and production cost. For example, in automotive body

assembly, a typical multi-station assembly process, the proportions of fixture-related dimensional faults during different production phases of a new product development areas high as 40% during pre-production, 70% during launch, 100% during one shift production and 70% during two shifts production<sup>1</sup>. A significant number of fixture failures are related to fixture installation and maintenance. For example, during pre-production, launch, one and two shifts production phases, 25%, 40%, 100% and 54% of the fixture-related faults are caused by discrepancies in fixture installation and maintenance<sup>1</sup>. From these data it can seen that accurate fixture installation and maintenance are very critical for overall product quality, and they have been reflected by extensive work focused on the (i) shortening ramp-up/launch of new products by fault root cause diagnosis<sup>2-8</sup>, (ii) reducing change over by rapid fixture deployment<sup>9,10</sup>, and (iii) increasing diagnosability by optimal sensor placement<sup>11-14</sup>. It is noticeable that dimensional fault diagnosis in manufacturing processes has aroused extensive interest of many researchers<sup>15</sup>. In a similar vein, Rong *et al*<sup>6</sup> proposed a diagnostic methodology for dimensional fault diagnosis of compliant beam structures. They obtained matrix  $\Gamma$  by using stiffness matrix of beam structures and applied least squares approach to estimate the faults in compliant assembly processes. In order to address the issue of ill-conditioning matrix  $\Gamma$ , Rong *et al*<sup>16</sup> presented an adjusted least squares approach which is able to overcome the ill-conditioning and give precise results for certain linear combinations of the faults.

In this article, according to the requirement of the angular variation in the process of aero crafts assembly, the precision analysis and control platform of digital precision assembly is built, and the dynamic controlling technology is studied<sup>17</sup>.

#### **Study on assembly accuracy controlling**

The typical aero crafts assembly process can be simplified as shown in the flow chart in Figure No., in which the flow direction stands for the assembly sequence. Through analysis, the assembly sequence can be regarded as the time sequence, the whole assembly process can be treated as a time-varying discrete systems. According to the state space theory,

the state observer model of assembly process and is established. Schematic diagram of the state observer model of assembly process is shown in Figure No.1 (the noise signal has been ignored in this Figure), the mathematical model is as follows,

$$\begin{cases} \tilde{X}_i = A_i \tilde{X}_{i-1} + B_i [u(i) + G_i C_i (\tilde{X}_{i-1} - X_{i-1})] + \xi_i \\ Y_i = C_i (\tilde{X}_{i-1} - X_{i-1}) + \eta_i \end{cases} \quad (1)$$

Where  $\tilde{X}_i$  represents the N dimension state vector of the actual product quality, which describes the actual assembly accuracy of products,  $X_i$  means the N dimension state vector of the theoretical product quality, which shows the theoretical assembly accuracy of products.  $A_i$  is the system matrix, means the mapping relation between station I and  $X_{i-1}$ , matrix  $B_i$  shows the impacts of station i on the assembly accuracy,  $u_i$  presents the assembly variation brought by station i,  $G_i$  shows the transition matrix of compensation value of station i-1,  $C_i$  is the

observation matrix, which stands for the number and the position of sensors.

Based on this model, according to the assembly process and the mapping relationship of assembly variation, the problem and the control of the compensation matrix  $G_i$  is studied, namely, the compensability of assembly variation.

After taking part and fixture error and reorientation-induced deviation in to consideration, the stream-of-variation model can be described in Figure No.2.

The stream-of-variation model can be characterized by the following equations:

$$\mathbf{X}(i) = \mathbf{A}(i-1)\mathbf{X}(i-1) + \mathbf{B}(i)\mathbf{U}(i) + \mathbf{W}(i) \quad i=1,2,\dots,N \quad (1)$$

$$\mathbf{Y}(i) = \mathbf{C}(i)\mathbf{X}(i) + \mathbf{V}(i) \quad i=1,2,\dots,N \quad (2)$$

The first equation, namely, the state equation shows that the cabin offset of station has two factors: the accumulated variation of station i-1 and the contribution variation of station i; the second equation is the output equation. System matrix  $A(i)$ ,  $B(i)$  and  $C(i)$  will give detailed description in this part.

$$\begin{bmatrix} \mathbf{X}(1) \\ \mathbf{X}(2) \\ \mathbf{X}(3) \\ \vdots \\ \mathbf{X}(N) \end{bmatrix} = \begin{bmatrix} \mathbf{B}(1) & 0 & 0 & \dots & 0 \\ \mathbf{A}(1)\mathbf{B}(1) & \mathbf{B}(2) & 0 & \dots & 0 \\ \mathbf{A}(2)\mathbf{A}(1)\mathbf{B}(1) & \mathbf{A}(2)\mathbf{B}(2) & \mathbf{B}(3) & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \mathbf{A}(N-1)\dots\mathbf{A}(1)\mathbf{B}(1) & \mathbf{A}(N-1)\dots\mathbf{A}(2)\mathbf{B}(2) & \mathbf{A}(N-1)\dots\mathbf{A}(3)\mathbf{B}(3) & \dots & \mathbf{B}(N) \end{bmatrix} \begin{bmatrix} \mathbf{U}(1) \\ \mathbf{U}(2) \\ \mathbf{U}(3) \\ \vdots \\ \mathbf{U}(N) \end{bmatrix} \quad (3)$$

Hence, the state vector  $X(i)$  updates as the change of stations:

$$\mathbf{X}(i) = \begin{bmatrix} X^1(i), \dots, X^{k_1}(i) | X^{k_1+1}(i), \dots, X^{k_2}(i) | X^{k_2+1}(i), \dots, X^{n_1}(i) \end{bmatrix}^T \quad (4)$$

Where  $n_1$  is the total number of cabins in assembly process.

**Modeling of variations**

At each station, the error sources are divided into two categories: positioning error and manufacture error of cabins. The positioning error causes the migration of cabin assembly. Manufacturing error refers to the cabin with feature size error and shape error in the manufacturing process. Effects of location error and fabrication error on the quality of the assembly are related, and modeling methods of each method are different, which is very adverse to study on the quality of the assembly effect. First of

all, the error is random; the variables must be based on statistical analysis. However, the results are not reliable if the variables are not independent; secondly, the manufacturing error and assembly error are very complex, and their model are not uniform, which gives analysis of inconvenience. In the study, based on the mode decomposition theory (frequency-domain analysis method), the manufacturing error and assembly error are decomposed into orthogonal error modes, so that the two expressions are unified, and the modes are independent of each other. In order to simplify the modeling process, there are two assumptions:

Smoothness assumption: the error field signal is sufficiently smooth; high frequency components (such as surface roughness and short wave error) can be ignored.

The height field hypothesis: the part feature error can be expressed by the height field function  $f(x, y)$  in the 2D region, and it is a stable random field process. The error sampling data  $f(n, m) = f(n\Delta x, m\Delta y)$  is transformed to the frequency domain, where  $n$  and  $m$  are expressed as sampling points along two axes. For the "smooth" error field, the error is mainly concentrated in the first few of the transform coefficients. Setting the number of sampling point is  $N^2$ , and the transform and inverse transform are as follows:

$$T(u, v) = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} f(n, m) g(n, m, u, v) \quad (5)$$

$$f(n, m) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} T(u, v) h(n, m, u, v) \quad (6)$$

Where  $T(u, v)$  represents the independent transformation parameters of error modes contributions  $u$  and  $v$  along X and Y axes.  $g(n, m, u, v)$  And  $h(n, n, u, v)$  are forward and inverse

$$g(n, m, u, v) = \frac{2}{N} \cos[2\pi nu / (2N - 1)] \cos[2\pi mv / (2N - 1)] \quad (8)$$

Among them,  $n, m = 0, 1, \dots, N - 1$ ,  $u, v = 0, 1, \dots, N - 1$   $N$  represent the number of samples and the number of modes. The inverse transform has the same form and the two dimensional transformation is as follows:

$$\begin{cases} C(0,0) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} f(n, m) \\ C(u, v) = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} f(n, m) g(n, m, u, v) \end{cases} \quad (9)$$

$$\begin{cases} f(0,0) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} C(u, v) \\ f(n, m) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} C(u, v) g(n, m, u, v) \end{cases} \quad (10)$$

The transform coefficients and the corresponding modes can easily explained the error fields of parts or sub assemblies. The super position approximation of the first 3 order modes of the formula expresses the rigid body mode of the shape error field. When  $u = v = 0$ :

transform kernels, and. The transformation can be considered as a series of rigid and deformable modal superposition, which describe the basic model of the error domain, as shown in Figure No.3. The coefficient  $T(u, v)$  represents the integer spatial frequency coefficients with  $u$  and  $V$  modes.

Each mode represents a different error mode, such as a rigid body mode (mode 1  $T(0,0)$ ) that represents the assembly location error, the deformation mode (mode 2 or 3,  $T(1,0), T(0,1)$ ) indicates the distortion (bending) deformation of the part.

The influence degree of each mode on the quality can be characterized by the amplitude of the parameter  $T(u, v)$ . A discrete cosine transform coefficient  $C(u, v)$  is established, and suppose that the error field signal function is  $f(n, m)$ , which can be obtained by measuring samples. The two dimensional transform cores is defined as:

$$g(n, m, 0, 0) = 1/N \quad (7)$$

$$f(n, m) |_{1st \text{ mod } e} = \frac{2}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} f(k, l) \quad (11)$$

The equation represents a horizontal plane (the rigid body motion of a part or subassembly), when  $u = 0, v = 1$  or  $u = 1, v = 0$ , the mode is the quasi rigid body mode, the high order mode has a more influence on the deformation of parts. The concept of mode decomposition of error domain is conducive to the separation and identification for different model error sources. For example, in a subassembly or product, rigid modal represents the deviation or variation generated by fixtures or parts. The deformation modal is the errors of parts or the deformation during the assembly process, such as assembly deformation, resilient, flexible interface and etc.

Regression analysis is used to estimate the statistical characteristics of  $C(u, v)$ , taking into account the form of continuous error domain function  $f(x, y)$ , using the following regression model:

$$f(x, y) = \sum_{u=0}^{\infty} \sum_{v=0}^{\infty} c_{u,v} \cos(ux\pi) \cos(vy\pi) \quad (12)$$

The continuous function basis  $\{h(x, y, u, v) = \cos(ux\pi) \cos(vy\pi) : u, v = 0, 1, \dots, \infty\}$  is orthogonal, representing the order of deformation

$$H_{u,v}^T = [h(1,0,u,v), h(1,1,u,v), h(1,2,u,v), \dots, h(1,N-1,u,v), h(2,0,u,v), h(2,1,u,v), \dots, h(2,N-1,u,v), \dots, h(N-1,N-1,u,v)]^T \quad (13)$$

$u, v = 0, 1, \dots, N-1, N \geq 1$  are also orthogonal,

$$(H_{u,v}^T)(H_{s,t}) = \begin{cases} N^2/4 & u = s \text{ and } v = t \\ 0 & u \neq s \text{ and } v \neq t \end{cases} \quad (14)$$

$$f(x, y) = c_0 + \sum_{u=1}^{I-1} \sum_{v=1}^{I-1} c_{u,v} \cos(ux\pi) \cos(vy\pi) + \varepsilon(x, y) = X\beta + \varepsilon(x, y) \quad (15)$$

Where the columns of  $\beta = [c_0, c_{1,1}, c_{1,2}, \dots, c_{I-1,I-1}]^T$  and  $X$  consist of a unit vector and an orthogonal basis vector ( $H_{u,v}$ ), which indicates the random error. The least squares estimates for all measurements  $f(n, m)$  are as follows:

$$\hat{f} = Xb \quad b = (X'X)^{-1} X'f \quad (16)$$

$$(X'X)^{-1} = 4/N^2[I] \quad (17)$$

Multiple regression theory is used to estimate. Suppose that  $\varepsilon(n, m) \sim N(0, \sigma^2)$ , under the Gauss and Markov assumptions,  $b$  is regarded as the unbiased estimation of  $\beta$ :

$$E(b) = \beta, \quad Cov(b) = \sigma^2(X'X)^{-1} = 4\sigma^2/N^2[I] \quad (18)$$

This shows the statistical independence of the model coefficients in  $b$ . If only the important part of  $b$  is reserved, and to judge the superiority of the estimation. Taking into account the orthogonality of

$$E[f(x, y)] = X\beta \quad Cov(f(x, y)) = E[(f - X\beta)(f - X\beta)'] = \sigma^2 I \quad (20)$$

$X_0 = (X_{00}, X_{01}, X_{02}, \dots, X_{0k})$ ,  $K < N$  Represent the value of  $k$  bases of random 1-L points. The prediction value of  $f(x, y)$  is  $\hat{f}_0 = X_0'b$ , from the Gauss and Markov assumptions

$$E[\hat{f}_0] = X_0'\beta,$$

$$var(\hat{f}_0) = X_0' cov(b) X_0 = 4\sigma^2/N^2[X_0'X_0] \quad (21)$$

If  $f_1, f_2, \dots, f_L$  allows the normal distribution:

modes, in which  $u, v$  represent the spatial frequency (the sample number of unit length). Similarly, discrete basis vectors:

According to the smooth ness assumption, the available basis functions can be used to fit the measured data:

each column, it can be proved that  $E(b_1) = \beta_1$ . In this case, this estimate is still unbiased.

Statistical significance tests for any of the independent regression parameters, such are constructed. For example,  $H_0 : \beta_j = 0; H_1 : \beta_j \neq 0$  can be used. The test statistics for this hypothesis are:

$$t_0 = \frac{b_j}{\sqrt{\hat{s}^2 a_{jj}}} = \frac{b_j N}{2\hat{s}} \quad (19)$$

Where  $a_{jj} = 4/N^2$  is the diagonal element of  $(X'X)^{-1}$ ? If  $|t_0| > t_{\alpha/2, N^2 - (I-1)^2}$ ,  $\beta_j$  is statistically significant. It is worth noting that, even if a parameter is statistically significant, it may still be eliminated from the model because of the small energy contribution. The rule of modal truncation is that only the parameters of the model are significant and the statistically significant can be kept.

The observation error in the model is estimated as

$$\hat{f}_0 - X_0'\beta \sim N(0, 4\sigma^2/N^2[X_0'X_0]) \quad (22)$$

The confidence interval of  $(1-\alpha) \times 100\%$  and the mean value of  $X_0'\beta$  are:

$$\hat{f}_0 \pm t_{r, \alpha/2} 2\hat{s} / N [X_0'X_0]^{1/2} \quad (23)$$

As for the observation value  $\hat{f}_0$ , the confidence interval or prediction interval for the observed value of  $(1-\alpha) \times 100\%$  is

$$\hat{f}_0 \pm t_{r,\alpha/2} s [1 + (4/N^2) X_0' X_0]^{1/2} \quad (24)$$

**Case study**

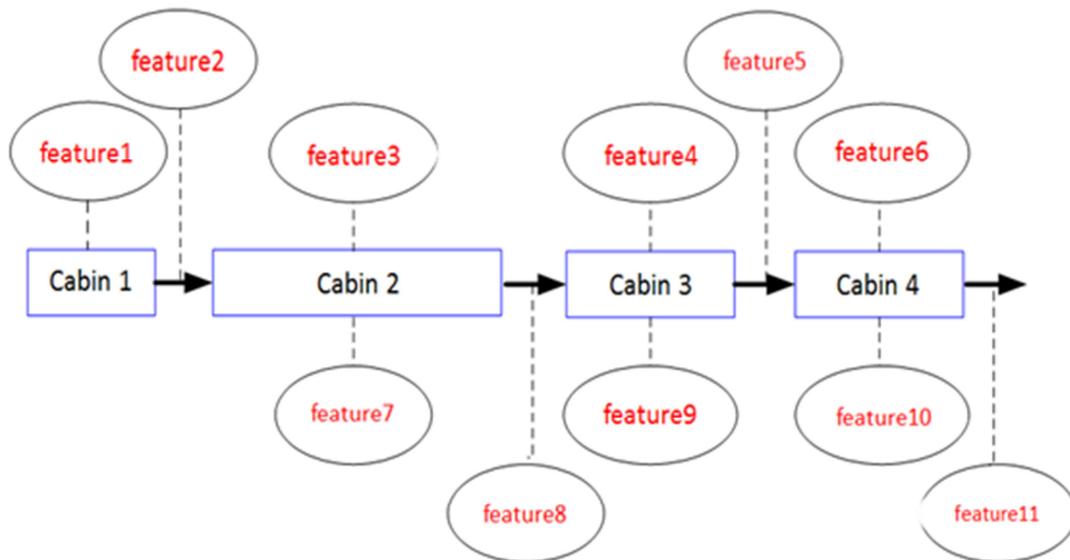
In this article, a three cabin test system is built, which is used for the verification of assembly error propagation model theory. The test object is divided into three sections of the cabin, as shown in Figure No.4.

First, the feature error of each cabin is measured with a laser tracker, such as the verticality error, flatness error of the connecting surface. The measured values are substituted into the state space model, still according to the cabin 1, cabin 2 and cabin 3 assembly sequences to calculate the offset of measuring points. Therefore, the angular variation between cabin 1 and cabin 3 along X axes can be calculated. The above process is repeated for 20 times. Then the actual measurement results are compared with the results of theoretical calculation, so we can get the correctness and trap of the theoretical method.

The overall test plan is shown in Figure No.5.

Through the measuring of mating feature of cabin parts, 6 parts of measuring data of 3 cabins are obtained.

The JBTEST function of MATLAB is used to test the normality of manufacturing error data of each characteristic plane. The statistic value jbstat of each set of data is less than the critical value, which is satisfied with the normal distribution. This shows that the normality assumption of reference point offset is credible when the simulation is carried out. The manufacturing error of each mating feature is put into the state space model, and then the angular variation of X direction is calculated. 24 sets of historical data are compared with the simulation results, which are shown in Figure No.7. From the fitting data of MATLAB and distribution diagram of simulation, we can see that their shape is similar. From the results, it can be seen that the calculated results are superior to that of actual measurement data, which is caused by the perfect prediction analysis to the actual situation; the main reason of the deviation is assembly stress release in placing and internal force of deformation.



**Figure No.1: The typical aero crafts assembly process**

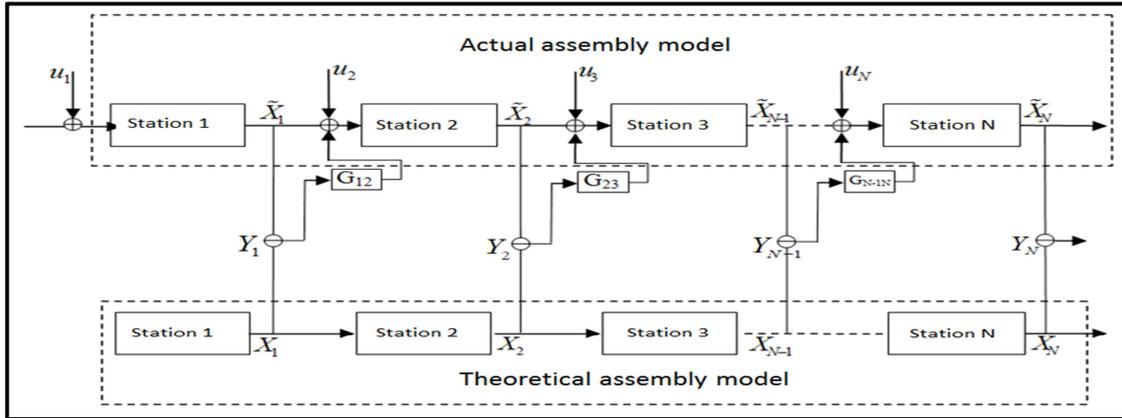


Figure No.1: The observation model of assembly process

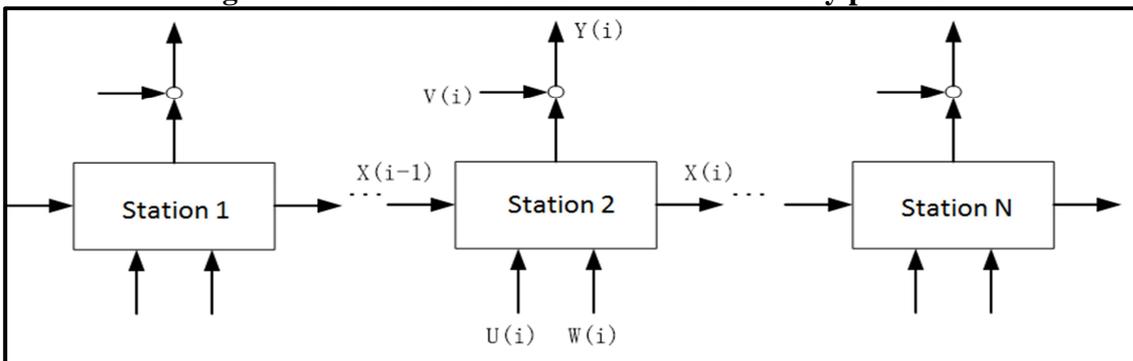


Figure No.2: The state space model of the assembly process

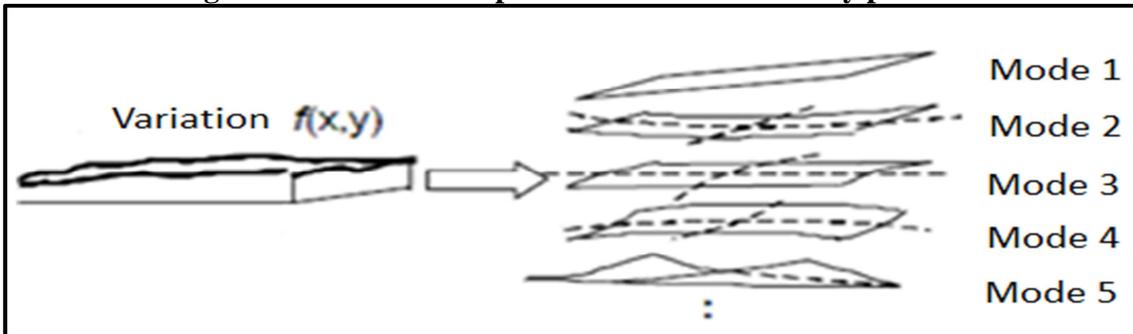


Figure No.3: The basic model of the error domain



a. Carbin 1

b. Carbin 2

c. Carbin 3

Figure No.4: The test system

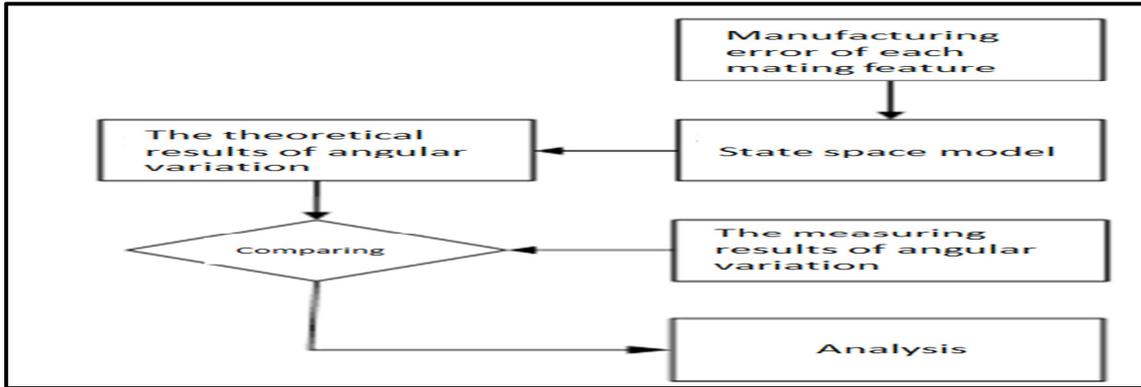


Figure No.5: The overall test plan

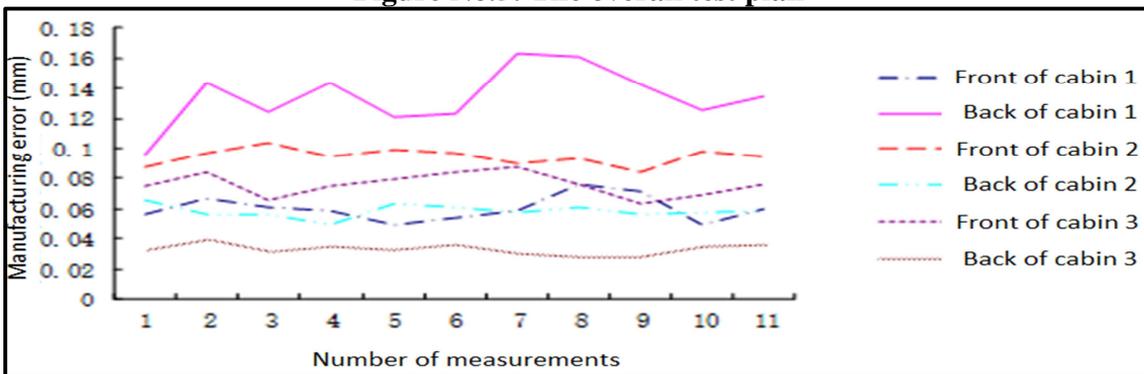


Figure No.6: The error of mating feature

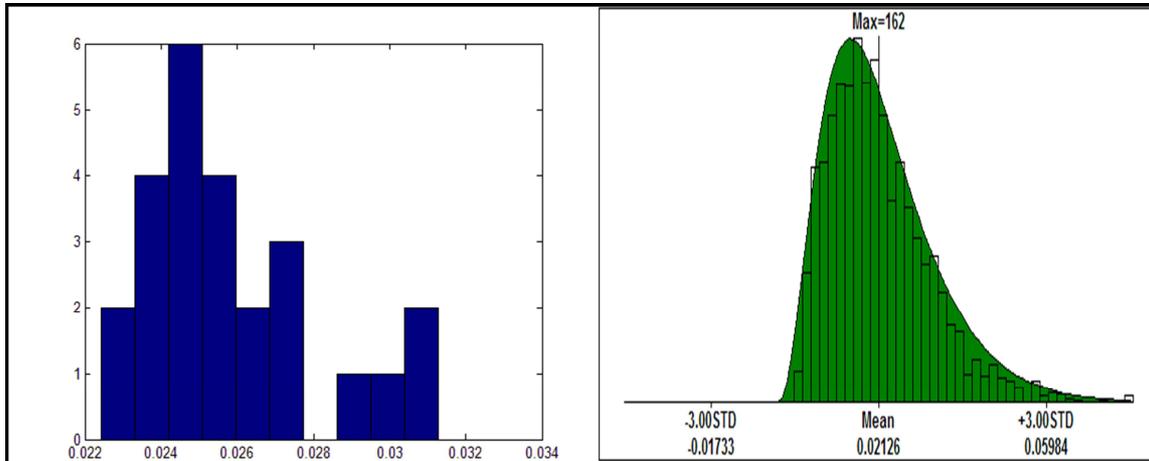


Figure No.7: The comparison diagram

**CONCLUSION**

The application research of digital assembly control technology in aero crafts assembly can break through many key technologies, such as error analysis, measurement, compensation techniques and etc. At the same time, through the analysis of the error modeling technology in the digital assembly system, we can grasp the digital assembly control technology

with three coordinate measuring instruments. This is of great significance for improving the level of the aero crafts assembly development, speeding up the application of advanced digital, automated assembly in aero crafts products.

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## CONFLICT OF INTEREST

We declare that have no conflict of interest.

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